Further Robustness Tests for the South African Anchovy and Sardine Resources, Including Maturity-at-Age

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Introduction

Cunningham and Butterworth (2004d) presented some results of robustness tests to the base case assessments of South African sardine and anchovy resources. A comparison between some summary statistics resulting when the resources were projected using OMP-04 was also made. However, a few of those results were still preliminary, given the poor convergence diagnostics for some of the MCMC chains simulated for the Bayesian analyses.

This document presents some final results for such robustness tests and also introduces some new robustness tests. Given recent work (van der Lingen 2004) suggesting that maturity-at-length may have changed over time, alternative maturity-at-age assumptions are also tested.

Anchovy Robustness Tests

The robustness tests to the base case anchovy assessment that are finalised in this document are:

A_0 – base case assessment (Cunningham and Butterworth 2004a)

A_{M1} – adult and juvenile natural mortality of 0.6 year^{-1}

A_R – Ricker stock-recruitment curve

For A_R, equation (A.5) of Cunningham and Butterworth (2004a) was replaced by:

$$N^A_{y,0} = a^A B^A_{y,N} e^{-b^A B^A_{y,N}} e^{\varepsilon^A_y}, \qquad y = 1980,\ldots,2002$$

and equation (A.9) was replaced by:

$$K^A = \frac{1}{b^A}\ln\left\{ a^A e^{\frac{1}{2}\left(0.4^2+\left(\lambda^A_0\right)^2\right)} \left[\sum_{a=1}^{4} \overline{w}^A_a e^{-M^A_{ju}-(a-1)M^A_{ad}}\right]\right\}.$$

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In addition, the prior distributions for the two stock-recruitment parameters in A_R were changed to $\ln(a) \sim U(-4,8)$ and $\ln\left(\frac{b^A}{1+b^A}\right) \sim U(-1000,1000)$.

Sardine Robustness Tests

The robustness tests to the base case sardine assessment that are finalised in this document are:

S_0 – base case assessment (Cunningham and Butterworth 2004b)

S_{kN1} – unbiased November spawner biomass surveys, i.e., $k_N^S = 1$. In the base case k_N^S was fixed at 0.72, indicating the survey underestimates the stock by a factor of 1.39, after a number of sources of error were taken into account, so that this test essentially considers the implications of the estimates for those correction factors having been too high.

Although the results for S_{slow}, a robustness test using an average age-length-key to represent a slower growth scenario for sardine, were preliminary in Cunningham and Butterworth (2004d), no further testing of this hypothesis will be carried out until some preliminary results on potential differences in the length at age are available.

Alternative Maturity Assumptions

In S_0, all sardine are assumed to mature at age 1. The following alternative maturity-at-age robustness tests were conducted:

S_{age2} – all sardine are assumed to mature at age 2

S_{ogive} – a maturity-at-age ogive is assumed

$S_{slowogive}$ – a maturity-at-age ogive is assumed, with a ‘slower growth’ assumption for years 2000 onwards.

The maturity-at-age ogives for each year used in S_{ogive} are given in Figure II.2 in Appendix II, together with a description of how the ogives were derived. The surprising result from the maturity-at-age ogives calculated is that a very small proportion of 1-year-olds are thought to be mature, and the proportion of 2-year-olds mature is very low. This is contrary to assumptions previously made. The only difference between S_{ogive} and $S_{slowogive}$ is that a different ogive (‘slow avg’ in Figure II.2c) is used in years 2000 to 2023 to represent a slower growth alternative.

Bayesian Integration

The AD Model Builder package was used to perform the Bayesian integration (Otter Research Ltd. 2000). As reported in Cunningham and Butterworth (2004d), a chain of 40 000 000 samples was simulated for the base case assessments, begun at the posterior mode. A burn-in of 15 000 000 was discarded and the remaining chain was thinned by 1000 to decrease any autocorrelation. The chain generated for A_{M1} was of the same length. A chain of length 500 000 000, was simulated for A_R; a burn-in of 200 000 000 was discarded and the remaining chain was thinned by 10 000 to decrease autocorrelation.

A number of further attempts have been made to obtain a converged MCMC chain for S_{kN1} since the preliminary results presented in Cunningham and Butterworth (2004d). Convergence has still not been obtained, but chains with improved convergence diagnostics over that used in Cunningham and Butterworth (2004d) have been obtained. The chain used to produce the relative summary statistics in this document was 40 000 000 long with a burn in of 5 000 and thinning of 1000. Mixing for the parameter $\left(\lambda_N^S\right)^2$ was improved by modifying the Hessian matrix.

The additional variance parameter $\left(\lambda_0^S\right)^2$ was estimated to be zero at the posterior mode for all the sardine robustness tests. When MCMC was run on these robustness tests, the convergence of the chain for $\left(\lambda_0^S\right)^2$ was severely hampered by slow mixing. Thus $\left(\lambda_0^S\right)^2$ was fixed at its posterior mode value of zero for all the MCMC runs of sardine robustness tests. A chain of length 150 million was run for S_{age2} and a burn-in of 37.5 million was required with the remaining samples being thinned by 2500. This gave 45 000 samples from which to calculate the marginal posterior distributions. A chain of length 40 million, with a burn-in of 15 million was run for S_{ogive}. The remaining samples were thinned by 1000 to decrease autocorrelation to give 25 000 samples. Even after $\left(\lambda_0^S\right)^2$ was fixed at its posterior mode value of zero for $S_{slowogive}$, convergence (according to the diagnostics used, see below) was not achieved for the MCMC chain for $\left(\lambda_N^S\right)^2$. A number of options involving much thinning and modifications to the Hessian matrix to improve mixing in the chain were attempted. The results presented in this document are from a chain of length 80 000 000, a burn-in of 40 000 000 and thinning of 2000, generated with the Hessian matrix from the mode modified to allow for bigger jumps over the parameter space of $\left(\lambda_N^S\right)^2$.

In order to more effectively compare these chains with fixed $\left(\lambda_0^S\right)^2 = 0$ to S_0, a further chain S_0^* was run in which $\left(\lambda_0^S\right)^2 = 0$ in S_0. Once again the Hessian matrix needed to be modified to allow for better mixing over the parameter space of $\left(\lambda_N^S\right)^2$. A chain of 30 000 000 was needed, with a burn-in of 5 000 000 and thinning of 1 000, giving a sample of 25 000 sets of parameters.

Convergence of the MCMC chains on the posterior distributions was tested using the BOA (Bayesian Output Analysis) package (Smith 2003). The diagnostics from the tests of Geweke (1992), Raftery and Lewis (1992) and Heidelberger and Welch (1983) were monitored and acceptable results were obtained for the above chains. In addition, the autocorrelations for each estimable parameter and cross-correlations between the parameters were also monitored to assess if further thinning or re-parameterisation was required. 500 sets of parameters were randomly sampled from the resultant chains for each robustness test, to be used in the input for OMP-04.

Results

Anchovy Robustness Tests

The results at the posterior mode are given in Tables 1 to 3 (repeated from Cunningham and Butterworth 2004d). For an initial comparison, the resource was projected forward using OMP-04 and the results at the posterior mode (Table 4; sardine base case MCMC results were used for these comparisons).

The resource was then projected forward using OMP-04 and the pertinent posterior distributions corresponding to these robustness tests, with the results presented in Table 4. Note that the risk for A_0 is now 0.28, even though OMP-04 was tuned for $Risk^A \leq 0.3$. This is because of the modification made to the exceptional circumstances provisions as documented in Cunningham and Butterworth (2004c), which results in a slightly lower risk for anchovy under the base case assessment.

From the summary statistics resulting from these projections, it is evident that, the risk for A_R, being 0.448, is the greatest (Table 4). However, this is a decrease from the provisional 0.474 presented in Cunningham and Butterworth (2004d). In this case the average biomass at the end of the projection period is estimated to be 29% of carrying capacity and down to 56% of the 2004 biomass. Were the anchovy resource to respond according to a Ricker stock-recruitment model, the expected average catch drops from 302 to 243 thousand tonnes.

The risk under the A_{M1} robustness test is less, although the average catch under OMP-04 would also be less. Hence this test does not warrant any concern for the implementation of OMP-04.

Sardine robustness tests

The results at the posterior mode are given in Tables 5 to 8 (S_0 and S_{kN1} repeated from Cunningham and Butterworth 2004d). When maturity was assumed to occur at age 2, the model fit to the data at the posterior mode does not differ substantially from the base case in which maturity was assumed to occur at age 1 (Table 5). However, the model fit to the data when a maturity ogive was assumed in S_{ogive} and $S_{slowogive}$ was less satisfactory than that achieved for S_0 (Table 5). This suggests that these ogives need careful discussion as to whether they may be biased. (Recall that the maturity ogives suggest a very low proportion of 1-year-olds to be mature, and further that the slower growth ogive suggests that a large proportion of even 4- and 5-year old sardine are immature, see Appendix II).

For an initial comparison, the resource was projected forward using OMP-04 and the results at the posterior mode (Table 9; anchovy base case MCMC results were used for these comparisons). The resource was projected forward using OMP-04 together with the pertinent posterior distributions. The results obtained using the posterior mode were broadly similar to the results obtained using samples from the posterior distributions in terms of higher and lower risk compared to S_0. Risk was higher under each of the alternative maturity assumptions compared to S_0, with the highest risk to the resource occurring under $S_{slowogive}$ (Table 9). However, the results for $S_{slowogive}$ may not be reliable since they were based on a chain for which convergence diagnostics were not fully satisfied. The summary statistics indicated a slightly lower risk for S_0^* compared to S_0, indicating there may be a slightly greater difference between the risk assumed for S_0 (upon which OMP-04 is based) and that calculated should an alternative maturity assumption hold true.

Due to the lack of convergence to the posterior distribution obtained for S_{kN1}, no summary statistics have been given in Table 9 in order to avoid any misleading conclusions being drawn. However, the results obtained thus far indicate that the risk to the resource would be greater than that under S_0, but not as large as that calculated to be the case under the alternative maturity assumptions.

Discussion

Previous robustness tests indicated that the risk to the sardine and anchovy resources would not differ substantially from that for operating models corresponding to the base case assessments of the resources (Cunningham and Butterworth 2004d). In this document, some further robustness tests have been considered, together with tests for which final results were not previously obtained.

The difference in the summary statistics resulting from projecting the population under OMP-04 and assuming a Ricker stock-recruitment curve for anchovy compared to the base case hockey-stick function are considerable. Given the available anchovy stock-recruitment data, it is impossible to *a priori* choose which stock-recruitment function best represents the South African anchovy resource. Therefore it is important that the potential higher risks to the resource under OMP-04, should the stock-recruitment dynamics follow a Ricker curve, be noted.

Projections using OMP-04 under alternative sardine robustness tests, and in particular, under alternative assumptions of sardine maturity-at-age, resulted in a greater risk to the resource than that calculated for the base case. As mentioned above, further discussions regarding the reliability of the calculated maturity-at-age ogives may weight any concern arising from this higher risk. However, pending these discussions and in the absence of further data to more accurately fix the assumed maturity-at-age in the base case assessment model, this potential higher risk should be noted.

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Table 1. Assumptions, likelihood and prior values for the anchovy robustness tests at the posterior mode. Blank cells indicate no change from A_0. (Symbols and headings are defined in Appendix I.)

Test	M^A_{ad}	M^A_{ja}	S-R curve	Ageing Method	k^A_g	$(\lambda^A_r)^2$	$(\lambda^A_N)^2$	Neg. Posterior	Neg. lnL	Neg. lnL_{Nov}	Neg. lnL_{Egg}	Neg. lnL_{Rec}	Neg. lnL_{Prop}	Neg. $lnPr(k_N)$	Neg. $lnPr(k_r)$	Neg. lnPr(del)	Neg. $lnPr(k_{prop})$
A_0	0.9	0.9	Hockey Stick	Prosch	1.0	estimated	fixed=0	45.86	20.62	-6.59	3.67	10.83	12.72	0.76	0.21	23.69	0.58
A_{M1}	0.6	0.6						50.43	27.02	-2.42	4.79	11.50	13.15	0.82	0.65	21.77	0.18
A_R			Ricker					42.61	21.45	-4.42	4.46	9.59	11.83	0.75	0.20	19.62	0.60

Table 2. Key model parameters for the anchovy robustness tests at the posterior mode. (Symbols and headings are defined in Appendix I.)

Test	k^A_N	k^A_r	k^A_r/k^A_N	k^A_q	$(\lambda^A_N)^2$	$(\lambda^A_r)^2$	$(\lambda^A_p)^2$	$(\lambda^A_0)^2$	$(\sigma^A_q)^2$
A_0	1.384	0.984	0.711	1.268	0.000	0.154	0.254	0.388	0.16
A_{M1}	1.416	1.315	0.929	0.945	0.000	0.167	0.283	0.292	0.16
A_R	1.380	0.975	0.706	1.281	0.000	0.131	0.201	0.184	0.16

Table 3. Key outputs from the anchovy robustness tests and key stock-recruitment parameters at the posterior mode (numbers in billions and biomass in thousands of tonnes). (Symbols and headings are defined in Appendix I.)

Test	$N^A_{2003,1}$	$N^A_{2003,2}$	$N^A_{2003,3}$	Average 84-99 Biomass	K^A	a^A	b^A	$\sqrt{0.4^2+(\lambda^A_0)^2}$	ε^A_{2002}	s^A_{cor}
A_0	131.8	45.6	62.7	1022.6	2306.6	227.7	461.3	0.740	0.877	0.565
A_{M1}	86.8	43.4	74.9	994.0	2492.3	145.9	498.5	0.672	0.812	0.548
A_R	141.7	50.9	61.5	1022.6	3158.7	0.3	0.0	0.587	0.465	0.288

Table 4. Summary statistics resulting from running OMP-04 under the anchovy robustness tests. Risk (the probability that adult anchovy biomass falls below 10% of the average adult anchovy biomass between November 1984 and November 1999 at least once during the projection period of 20 years), $Risk^A$*, average directed catch (in thousands of tonnes),* $\overline{C}^A$*, average proportional annual change in directed catch,* AAV^A*, average biomass at the end of the projection period as a proportion of carrying capacity, as a proportion of the risk threshold, and as a proportion of biomass at the beginning of the projection period, and average minimum biomass over the projection period as a proportion of carrying capacity and as a proportion of the risk threshold, for the OMP-04 trade-off point are reported. Results are presented using anchovy results from the posterior mode only and from the posterior distributions obtained using MCMC.*

	Posterior Mode Only			Posterior Distributions		
	A_0	A_{M1}	A_R	A_0	A_{M1}	A_R
$Risk^A$	0.072	0.096	0.180	0.280	0.228	0.448
$\overline{C}^A$	333.1	323.2	291.1	302.3	284.9	242.8
AAV^A	0.273	0.286	0.285	0.334	0.342	0.377
$\overline{B^A_{2023}/K^A}$	0.675	0.550	0.324	0.686	0.596	0.292
$\overline{B^A_{2023}/BRisk^A}$	1.523	1.379	1.002	1.502	1.465	0.930
$\overline{B^A_{2023}/B^A_{2004}}$	0.002	0.001	0.001	1.010	0.891	0.555
$\overline{B^A_{\min}/K^A}$	0.188	0.164	0.117	0.134	0.127	0.072
$\overline{B^A_{\min}/BRisk^A}$	0.424	0.412	0.361	0.279	0.302	0.225

Table 5. Assumptions, likelihood and prior values for the sardine robustness tests at the posterior mode. Blank cells indicate no change from S_0.. (Symbols and headings are defined in Appendix I.)

Test	M^S_{ad}	M^S_{ju}	S-R curve	k^S_N	$(\lambda^S_r)^2$	$(\lambda^S_N)^2$	Mat-urity	Neg. Posterior	Neg. lnL	Neg. lnL_{Nov}	Neg. lnL_{Rec}	Neg. lnL_{Prop}	Neg. $lnPr(k_r)$	Neg. lnPr(del)	Neg. $lnPr(k_{prop})$	Neg. $lnPr(var_{prop})$	Neg. lnPr(a)	Neg. $lnPr(a_2)$
S_0	0.4	1.0	Hockey Stick	0.7195	Esti-mated	Esti-mated	Age 1	69.44	47.17	1.42	14.75	30.99	0.61	7.42	4.50	3.08	5.44	1.23
S_{kN1}				1.0				71.16	48.67	2.17	14.56	31.94	0.93	7.32	4.66	3.14	5.24	1.20
S_{age2}							Age 2	71.29	48.14	1.92	14.93	31.29	0.39	8.05	4.37	3.09	5.75	1.49
S_{ogive}							Ogive	77.86	52.66	6.32	17.75	28.60	0.36	10.49	4.41	2.88	5.90	1.15
$S_{slowogive}$							Ogive	87.22	62.71	15.16	20.17	27.39	0.44	9.85	4.58	2.79	5.94	0.91

Table 6. Key model parameters for the sardine robustness tests at the posterior mode. (Symbols and headings are defined in Appendix I.)

Test	k_N^S	k_r^S	k_N^S / k_r^S	$k_{p,1}^S$	$k_{p,2}^S$	$k_{p,3}^S$	$k_{p,4}^S$	$k_{p,5}^S$	$(\lambda_N^S)^2$	$(\lambda_r^S)^2$	$(\lambda_0^S)^2$	$(\sigma_q^S)^2$
S_0	0.720	1.045	1.453	1.189	0.781	1.043	0.884	1.006	0.000	0.230	0.000	6.582
S_{kN1}	1.000	1.331	0.751	1.168	0.771	1.068	0.937	1.115	0.000	0.222	0.009	6.742
S_{age2}	0.720	0.804	1.117	1.203	0.783	1.021	0.841	0.928	0.000	0.237	0.000	6.616
S_{ogive}	0.720	0.500	0.695	1.182	0.792	1.034	0.850	0.961	0.021	0.369	0.000	6.081
$S_{slowogive}$	0.720	0.430	0.598	1.148	0.795	1.067	0.910	1.063	0.184	0.543	0.000	5.856

Table 7. Key outputs from the sardine robustness tests at the posterior mode (numbers in billions and biomass in thousands of tonnes). (Symbols and headings are defined in Appendix I.)

Test	$N_{2003,1}^S$	$N_{2003,2}^S$	$N_{2003,3}^S$	$N_{2003,4}^S$	Average 91-94 Biomass	S_1	S_2	S_3	S_4
S_0	31.0	22.6	15.7	7.9	898.1	0.648	1.000	0.865	0.342
S_{kN1}	23.3	16.6	11.4	5.7	662.6	0.645	1.000	0.892	0.362
S_{age2}	44.5	27.2	22.2	10.3	875.1	0.681	1.000	0.832	0.315
S_{ogive}	55.42	37.89	27.45	18.02	865.8	0.783	1.000	0.745	0.265
$S_{slowogive}$	56.88	38.14	26.33	17.26	1007.9	0.894	1.000	0.689	0.225

Table 8. Key stock-recruitment parameters and outputs for the sardine robustness tests at the posterior mode. (Symbols and headings are defined in Appendix I.)

Test	K^S	a^S	$a_{1979-1983}^S$	b^S	$\sqrt{0.4^2 + (\lambda_0^S)^2}$	ε_{2002}^S	s_{cor}^S
S_0	6267.0	91.811	3.273	2569.6	0.400	-0.037	0.236
S_{kN1}	4891.4	71.352	3.076	1953.1	0.411	-0.062	0.232
S_{age2}	8378.0	122.736	3.638	2700.3	0.400	0.091	0.281
S_{ogive}	9452.7	138.481	3.797	1654.3	0.400	0.312	0.188
$S_{slowogive}$	9731.7	142.568	3.694	1386.5	0.400	0.302	0.219

Table 9. Summary statistics resulting from running OMP-04 under the sardine robustness tests. Risk (the probability that adult sardine biomass falls below the average adult sardine biomass between November 1991 and November 1994 at least once during the projection period of 20 years), $Risk^S$, average directed catch (in thousands of tonnes), $\overline{C}^S$, average proportional annual change in directed catch, AAV^S, average biomass at the end of the projection period as a proportion of carrying capacity, K, as a proportion of the risk threshold, BRisk, and as a proportion of biomass at the beginning of the projection period, and average minimum biomass over the projection period as a proportion of carrying capacity and as a proportion of the risk threshold, for the OMP-04 trade-off point are reported. Results are presented using sardine results from the posterior mode only and from the posterior distributions obtained using MCMC. Results have not been given for SkN1 due to the lack of convergence to the posterior distribution obtained for the MCMC chain (see page 3).

	Posterior Mode Only					Posterior Distributions					
	S_0	S_{kN1}	S_{age2}	S_{ogive}	$S_{slowogive}$	S_0	S_{kN1}	S_0^*	S_{age2}	S_{ogive}	$S_{slowogive}^{\#}$
$Risk^S$	0.020	0.046	0.014	0.058	0.988	0.096	↑	0.072	0.372	0.232	0.388
$\overline{C}^S$	373.1	367.0	390.2	308.4	199.5	365.9	↑	428.6	361.3	381.7	339.7
AAV^S	0.214	0.232	0.197	0.284	0.351	0.197	↓	0.128	0.190	0.194	0.280
$\overline{B^S_{2023}/K^S}$	0.771	0.687	0.626	0.367	0.153	0.728	↓	0.735	0.459	0.353	0.140
$\overline{B^S_{2023}/BRisk^S}$	3.586	3.382	3.999	2.668	0.988	4.009	↑	4.612	2.630	3.399	2.328
$\overline{B^S_{2023}/B^S_{2004}}$	0.565	0.542	0.645	0.548	0.533	0.643	↑	0.697	0.599	0.655	0.642
$\overline{B^S_{min}/K^S}$	0.496	0.430	0.370	0.216	0.090	0.451	↓	0.463	0.267	0.208	0.083
$\overline{B^S_{min}/BRisk^S}$	2.309	2.114	2.364	1.576	0.580	2.445	↑	2.771	1.486	1.986	1.333

These results were obtained from a sample of a chain that had not fully converged for all parameters (see pg 3).

Appendix I: Glossary of Terms Used in Tables

$M_{ju}^{A/S}$ - rate of natural mortality (in year[-1]) of juvenile anchovy/sardine (i.e. fish of age 0).

$M_{ad}^{A/S}$ - rate of natural mortality (in year[-1]) of adult anchovy/sardine (i.e. fish of age 1+).

k_g^A - constant of proportionality (multiplicative bias) in the November egg survey estimate of spawner biomass.

$k_N^{A/S}$ - constant of proportionality (multiplicative bias) in November acoustic survey estimate of spawner biomass.

$k_r^{A/S}$ - constant of proportionality (multiplicative bias) in the acoustic survey estimate of recruitment.

$k_{p,a}^S$ - constant of proportionality (multiplicative bias) in the estimate of the proportion (by number) of sardine of age a in the November survey.

k_q^A - is a multiplicative bias associated with the proportion of 1-year-olds in the November survey.

$\left(\lambda_r^{A/S}\right)^2$ - the additional variance (over and above the survey sampling CV that reflects survey inter-transect variance) associated with the recruit surveys.

$\left(\lambda_N^{A/S}\right)^2$ - the additional variance (over and above the survey sampling CV that reflects survey inter-transect variance) associated with the November surveys.

$\left(\lambda_p^A\right)^2$ - the additional variance (over and above the fixed variance of 0.4^2) associated with fitting the proportion of anchovy 1-year-olds in the November survey.

$\left(\lambda_0^{A/S}\right)^2$ - the additional variance (over and above the fixed variance of 0.4^2) associated with the recruitment residuals.

$(\sigma_p^S)^2$ - the overall variance-related parameter for the log-transformed sardine proportion-at-age observations, $p_{y,a,Nov}^S$ [note variance = $(\sigma_p^S)^2 / (n_y\, p_{y,a,Nov}^S)$].

$\left(\sigma_q^A\right)^2$ - a minimum variance associated with the proportion of anchovy 1-year-olds in the likelihood.

Neg. Posterior - negative posterior (negative log-likelihood * negative log joint prior)

Neg. lnL - negative log-likelihood.

$\ln L_{Nov}$ - portion of the log-likelihood from fitting to the November acoustic survey estimates.

$\ln L_{Egg}$ - portion of the log-likelihood from fitting to the November egg survey estimates.

$\ln L_{Rec}$ - portion of the log-likelihood from fitting to the recruitment survey estimates.

$\ln L_{Prop}$ - portion of the log-likelihood from fitting to the proportion-at-age in the November survey lnPr(k_N) - log prior of k_N^A (anchovy only).

lnPr(k_r) - log prior of k_r^A (anchovy only).

lnPr(del) - log joint prior of the recruitment residuals.

lnPr(k_{prop}) - log prior of $k_{p,a}^S$ (sardine) or k_q^A (anchovy).

lnPr(var_{prop}) - log prior of the variance in the proportion-at-age.

lnPr(a) - log prior of $a^{A/S}$.

lnPr(a_2) - log prior of $a_{1979-1983}^S$ (sardine only).

$N_{2003,1}^{A/S}$ - number (in billions) of anchovy/sardine of age *a* at the beginning of November 2003.

S_a - recent sardine fishing selectivities-at-age.

$K^{A/S}$ - carrying capacity.

$a^{A/S}$ - maximum recruitment (in billions) in the hockey stick stock-recruitment curve (see pg 2 and 3 for definitions for other stock-recruitment curves).

$a_{1979-1983}^S$ - maximum recruitment (in billions) in the hockey stick stock-recruitment curve for 1979 to 1983.

$b^{A/S}$ - spawner biomass above which there should be no recruitment failure risk in the hockey stick stock-recruitment curve (see page 2 and 3 for definitions for other stock-recruitment curves).

$\sqrt{0.4^2 + \left(\lambda_0^{A/S}\right)^2}$ - standard deviation in recruitment residuals.

$\varepsilon_{2002}^{A/S}$ - lognormal deviation of anchovy/sardine recruitment in 2002.

$s_{cor}^{A/S}$ - recruitment serial correlation.

Appendix II: Calculation of Maturity-at-Age Ogives

Van der Lingen (2004) recently provided maturity-at-length ogives for sardine. The ogive was only measured up to 23.5cm for 1988-1995 and to 22cm for 1996-2003. A linear extension of this ogive was made to 25cm, whereafter the sardine were assumed to be 100% mature (T. Fairweather, C. van der Lingen pers. comm.) These ogives are given in Figure II.1. Age-length-keys from the November spawner biomass surveys from 1988 to 1999 were used to convert these ogives into maturity-at-age ogives. This appendix describes the calculations and assumptions used in this process.

Let:

$ALK_{a,l}$ - denote the proportion of fish sampled from length class l that were aged a years old, in the November survey;

N_l^{sample} - denote the number of fish sampled in length class l; and

mat_l - denote the proportion of fish from length class l that are mature (Figure II.1).

Then the numbers-at-age by length class were calculated by:

$N_{a,l} = ALK_{a,l} \times N_l^{sample}$, $a = 1,...,5$

The numbers mature-at-age by length class could simply be calculated by

$Nmat_{a,l} = mat_l \times N_{a,l}$, $a = 1,...,5$

and the proportion mature-at-age therefore

$mat_a = \sum_l Nmat_{a,l} \Big/ \sum_l N_{a,l}$, $a = 1,...,5$.

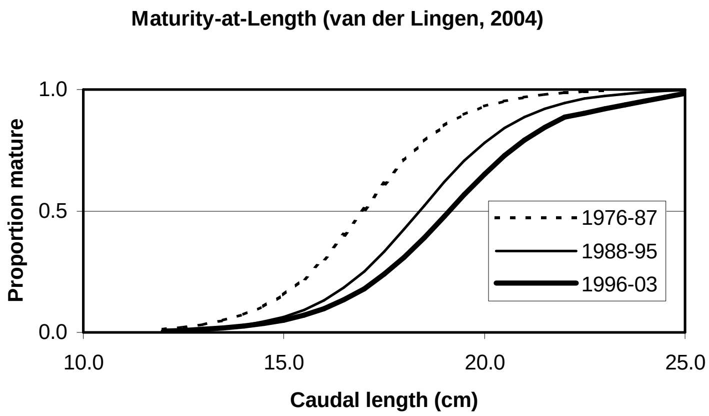


Figure II.1. Maturity-at-length ogives for sardine.

However, the above method would imply in a case where 50% in length class l have been measured to be mature, and the numbers-at-age are split with 20% aged 2, 40% aged 3 and 40% aged 4, that 50% of the 2-year-olds, 50% of the 3-year-olds and 50% of the 4-year-olds would be mature, while the remaining fish would be immature. A more intuitive method would result in 100% of the 4-year-olds being mature (being 40% of the fish), 25% of the 3-year-olds being mature (being 10% of the fish) and 0% of the 2-year-olds being mature. This method required the following calculations:

$$Nmat_{0,l} = \begin{cases} \max\left(0, mat_l \times N_l^{sample} - \sum_{a=1}^{5} Nmat_{a,l}\right) & mat_l \times N_l^{sample} < \sum_{a=0}^{5} N_{a,l} \\ N_{0,l} & mat_l \times N_l^{sample} \geq \sum_{a=0}^{5} N_{a,l} \end{cases}$$

$$Nmat_{1,l} = \begin{cases} \max\left(0, mat_l \times N_l^{sample} - \sum_{a=2}^{5} Nmat_{a,l}\right) & mat_l \times N_l^{sample} < \sum_{a=1}^{5} N_{a,l} \\ N_{1,l} & mat_l \times N_l^{sample} \geq \sum_{a=1}^{5} N_{a,l} \end{cases}$$

$$Nmat_{2,l} = \begin{cases} \max\left(0, mat_l \times N_l^{sample} - \sum_{a=3}^{5} Nmat_{a,l}\right) & mat_l \times N_l^{sample} < \sum_{a=2}^{5} N_{a,l} \\ N_{2,l} & mat_l \times N_l^{sample} \geq \sum_{a=2}^{5} N_{a,l} \end{cases}$$

$$Nmat_{3,l} = \begin{cases} \max\left(0, mat_l \times N_l^{sample} - \sum_{a=4}^{5} Nmat_{a,l}\right) & mat_l \times N_l^{sample} < \sum_{a=3}^{5} N_{a,l} \\ N_{3,l} & mat_l \times N_l^{sample} \geq \sum_{a=3}^{5} N_{a,l} \end{cases}$$

$$Nmat_{4,l} = \begin{cases} \max\left(0, mat_l \times N_l^{sample} - Nmat_{5,l}\right) & mat_l \times N_l^{sample} < \sum_{a=4}^{5} N_{a,l} \\ N_{4,l} & mat_l \times N_l^{sample} \geq \sum_{a=4}^{5} N_{a,l} \end{cases}$$

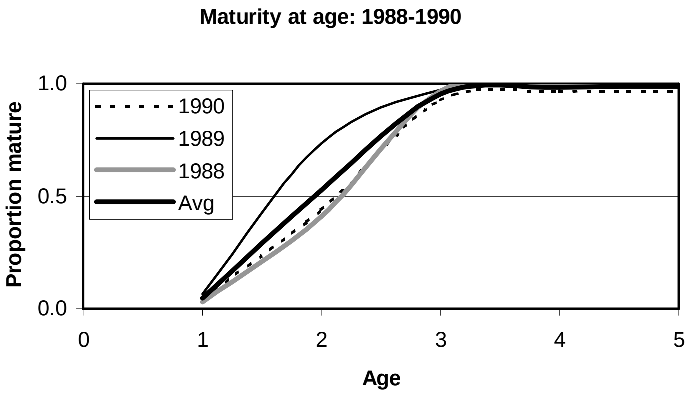
$$Nmat_{5,l} = \begin{cases} \max\left(0, mat_l \times N_l^{sample}\right) & mat_l \times N_l^{sample} < N_{5,l} \\ N_{5,l} & mat_l \times N_l^{sample} \geq N_{5,l} \end{cases}$$

No age-length-keys were available for 2000-2003 and so N_l^{sample} is unavailable for these years. Similarly for 1984-1987, N_l^{sample} is unavailable, even though some age-length-keys do exist for these years. In these years the above method of calculating maturity-at-age ogives could not be used.

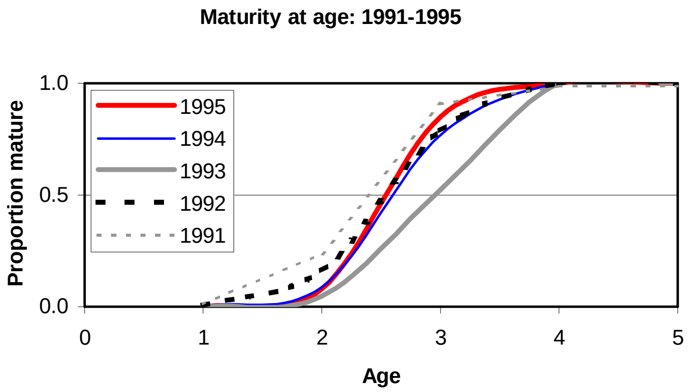
The maturity-at-length ogive calculated by van der Lingen (2004) for 1976-1987 differed from that calculated from 1988 to 1995, with the former measuring a greater proportion mature at smaller length classes (Figure II.1). The maturity-at-age ogives calculated using the above method for 1988 to 1995 (Figure II.2a&b) indicate a trend in a smaller proportion being mature for the same age class as time progressed from 1988 to 1995. The maturity-at-age ogive for 1984 to 1987 was therefore based on an average ogive from 1988 to 1990 (labelled ‘Avg’ in Figure II.2a).

Similarly, the maturity-at-age ogives for 1996 to 1999 were used to calculate an average ogive to be used for 2000 to 2003 as a base-case option. An alternative maturity-at-age ogive (labelled 'Slow Avg' in Figure II.2c) was also calculated to mimic a slower growth option in the same manner as the slower growth robustness test, S_{slow}, was done. This involved calculating slower growth maturity-at-age ogives for 1996 to 1999 and calculating an average from the resultant ogives to be used for 2000 to 2003.

a)



b)



c)

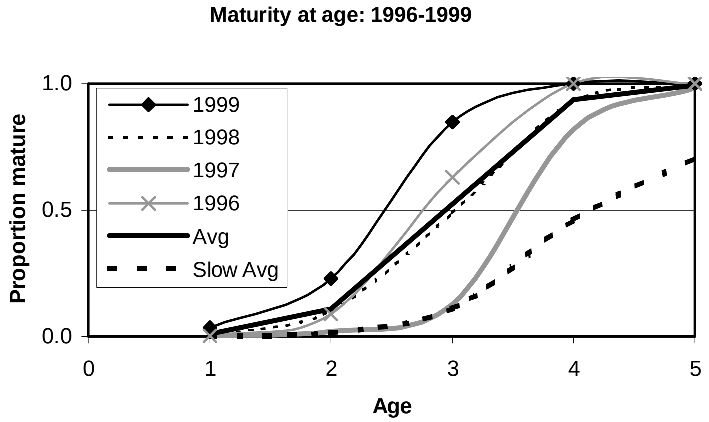


Figure II.2. Maturity-at-age ogives for sardine. In a) the average curve is calculated from 1988 to 1990 and is used to represent the ogive for 1984 to 1987. In c) the average curve is calculated from 1996 to 1999 and is used to represent the ogive from 2000 to 2023 and the slower growth average curve is calculated from 'slower growth' ogives for 1996 to 1999 and is used to represent a 'slower growth' alternative from 2000 to 2023.